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Value-at-Risk Based Portfolio Allocation Using Particle Swarm Optimization

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***Abstract*—Risks and returns are inevitably interlinked in today's work-a-day real world financial transactions. In particular, a financial portfolio illustrates the situation in which a combination of financial instruments/assets describes this interrelation in terms of their correlation in a particular market condition. The field of portfolio management has assumed importance of late, thanks to the need for decision making in investment opportunities in a high-risk scenario. It addresses the risk-reward tradeoff allocation of investments to a number of different assets so as to maximize returns or minimize risks in a given investment period. In this paper, a particle swarm optimization procedure is used to evolve optimized portfolio asset allocations in a volatile market condition. The proposed approach is centered around optimizing the Value-at-Risk (*VaR*) measure in different market conditions based on several objectives and constraints. Applications of the proposed approach are demonstrated on a collection of several financial instruments.**

Keywords: Portfolio Management; Financial Instruments; Value-at-Risk; Particle Swarm Optimization

I. INTRODUCTION

Risks and returns are inevitably interlinked in today's work-a-day real world financial transactions. In particular, a financial portfolio illustrates the situation in which a combination of financial instruments/assets describes this interrelation in terms of their correlation in a particular market condition. The field of portfolio management has assumed importance of late, thanks to the need for decision making in investment opportunities in a high-risk scenario. It addresses the risk-reward tradeoff allocation of investments to a number of different assets so as to maximize returns or minimize risks in a given investment period. Markowitz [1] recommended that an investor should not select his/her assets due to only characteristic features, but he/she needs to consider how each asset has co-moved with all other assets at hand. Markowitz quantified risk as the standard deviation of returns. He showed how the diversification into several investments that have limited or no positive correlation in their movements can reduce overall risk. According to Markowitz, this movement is measured by a correlation coefficient varying between +1 and -1. From the properties of correlation coefficient therefore, two investments with a positive correlation will move in lock-step forward with one another, while those with a negative correlation will move in exactly in the opposite direction. As the correlation coefficient is a quantitative measure of the variance of a portfolio, any coefficient less than +1 will reduce the overall variance of that portfolio.

Subject to the same expected yield/return, if these co-movements are taken into account [2], an investor can construct a portfolio having a lesser risk than a portfolio constructed without paying any heed to the interaction between securities. This model was later modified by Black [3] to allow short-selling of assets or negative weights of assets, thereby forming a closed form solution to the problem.

A plethora of approaches in the lines of risk management to solve the portfolio optimization problem are reported in the literature [4] [5]. Most of these techniques mainly incorporate a combination of portfolio states thereby rendering the problem intractable and time-complex. Moreover, these techniques suffer from their inability to tackle the underlying nonlinearities in the objectives and constraints in the problem. Mokhtar *et al*. presented a review of mathematical programming models for portfolio optimization in [6]. A new family of estimators of the covariance matrix that relies solely on forward-looking information of the assets for portfolio optimization is introduced in [7].

Value-at-Risk (*VaR*) [8] [9] [10] is a measure of how the market value of an asset or of a portfolio of assets is likely to decrease over a certain time period (usually over 1 day or 10 days) under typical market conditions. The power of *VaR* lies in its generality. Unlike market risk metrics such as the Greeks, duration, convexity or beta, which are applicable to only certain asset categories or certain sources of market risks, *VaR* is common and general. It is based on the probability distribution function (PDF) for a portfolio's market value. All liquid assets have uncertain market values, which can be characterized with probability distributions. All sources of market risk contribute to those probability distributions. Being applicable to all liquid assets and encompassing, at least in theory, all sources of market risk, *VaR* is a broad metric of market risk.

The generality of *VaR* poses a computational challenge. In order to measure market risk in a portfolio using *VaR*, some means must be found for determining the probability distribution of that portfolio's market value. Obviously, the more complex a portfolio is, the more asset categories and sources of market risk it is exposed to, i.e. the more challenging that task becomes. A *VaR* calculates an amount of money, measured in that currency, such that there is that probability of the portfolio not loosing that amount of money over that horizon. In the terminology of mathematics, this is called a quantile, so one-day 90% USD *VaR* is just the .90-quantile of a portfolio's one-day loss.

To be precise, *VaR* is a quantile of loss. The task of a value-at-risk measure is to calculate such a quantile. Banks, broker dealers and investment banks use *VaR* to measure the market risk of their proprietary owned assets. *VaR* varies widely depending on the conditions, asset class, historical performance, volatility/standard deviation, downside risk and expected shortfall. The *VaR* approach to risk management aims to consolidate in a consistent way, at the organization or entity level, the risks inherent in a portfolio of various classes of financial instruments. The results are expressed as a single number - the *VaR* i.e., in terms of the maximum expected loss, the confidence interval of the loss and the number of days in the risk period.

In this paper, a novel approach for achieving an optimized solution to the portfolio evolution problem is presented. The proposed approach is centered around the optimization of the *VaR* measures of a portfolio comprising several financial instruments in different market conditions based on several objectives and constraints. Applications of a particle swarm optimization based *VaR* optimization procedure are demonstrated with reference to the minimization of the risks involved in the portfolio under consideration, thereby minimizing the portfolio loss incurred.

Section II of the paper provides an overview of the Value-at-Risk (*VaR*) measure used for portfolio risk management analysis. Section III elucidates the mathematical formulation of the *VaR* measure. The different models for the computation of *VaR* are discussed in Section IV. A brief description of the particle swarm optimization procedure along with the associated algorithm is illustrated in Section V. The results of the findings are summarized in Section VI. Finally, Section VII draws a line of conclusion with future directions of research.

II. VALUE-AT-RISK (*VAR*)

In economics and finance, *VaR* is defined as the maximum loss which does not exceed with a given probability (the confidence level) over a given period of time or horizon. *VaR* is most commonly used by security firms or investment banks to measure the market risk of their asset portfolios (market value at risk). It is widely applied in finance for quantitative risk management for many types of risk. However, it might be noted at this point that *VaR* does not give any information about the severity of loss by which it exceeds.

Estimation of three parameters is required for the determination of *VaR*. These are (i) the time horizon (period) to be analyzed, which relates to the time period over which a financial institution is committed to holding its portfolio, or to the time required to liquidate assets. Typical time horizons are 1 day, 10 days, or 1 year, (ii) the confidence level, which is the interval estimate in which the *VaR* would not be expected to exceed the maximum loss. Commonly used confidence levels are 99% and 95%. However, confidence levels are not indications of probabilities, and (iii) the unit of *VaR*, which is given in a unit of the currency.

Given a probability p and K days, where p and K must be predetermined by the risk manager, *VaR* is defined as a number such that there is a probability p of exhibiting a worse return over the next K days. *VaR* is thus simply a quantile of the return distribution and thus does not reflect anything about the risk distribution. More important, it does not indicate as to how large the likely magnitude of losses is on those days when the return is worse than the *VaR*. On the other hand, Expected Shortfall (*ES*), which is defined as the expected return conditional on the return being worse than the *VaR*, has been suggested as an alternative to *VaR*. Needless to state, in spite of all these limitations, *VaR* still remains to be the most common risk metric used in practice.

A *VaR* enabled given portfolio asset allocation model should posses the following characteristics:

1. The asset allocation model should be a fully specified data-generating and data-intensive one which can be estimated and implemented on daily returns for portfolios with a large number of varied assets.
2. It should allow the computation of *VaR* for any prespecified level of confidence (p) and for any horizon of interest (K) subject to the current market conditions.
3. It should also be flexible enough to allow calculation of risk measures other than *VaR*. The model should reflect the following itemized facts of daily asset returns in order to deliver accurate risk predictions:
   - Daily returns should have little or no exploitable conditional mean predictability. The variance of daily returns should be predictable and should greatly exceed the mean return.
   - Daily returns are not normally distributed. Even after standardizing daily returns by a dynamic variance model, the standardized daily returns are not normally distributed. Positive and negative returns of the same magnitude may have different impacts on the variance.
4. The correlations between assets should appear to be time-varying.
5. As the investment horizon increases, the return data distribution should approach the normal distribution.

Thus, given these salient features of daily asset returns, a portfolio optimization technique involving the *VaR* reduces to building a dynamic market risk management model that contains only few parameters to be estimated, and that is easily implemented on a large set of assets.

III. MATHEMATICAL FORMULATION

As already stated, *VaR* is an important measure of the exposure of a given portfolio to different kinds of risk inherent in financial environments, which can be used for portfolio optimization purposes.

Given a portfolio P composed of k assets $S = \{S_1, S_2 \ldots S_k\}$, and $W = \{W_1, W_2 \ldots W_k\}$ the relative weights or portions of the assets in the portfolio, the price of the portfolio at time t is given by

$$P(t) = \sum_{i=1}^{k} S_i(t) W_i \qquad (1)$$

where, $S_i(t)$ and W_i are the value and importance level of the portfolio at time t, respectively.

The *VaR* of the portfolio P, which is the maximum expected loss over a holding period at a given level of confidence (α), can then be defined as the smallest number l such that the probability by which the loss L exceeds l is not larger than (1 - α), i.e.,

$$VaR_\alpha = \inf\{l \in R : P(L > l) \le 1 - \alpha\} = \inf\{l \in R : F_L(l) \ge \alpha\} \qquad (2)$$

IV. COMMON VAR CALCULATION MODELS

A plethora of techniques and models for estimating *VaR* from the time horizon, confidence level and the unit of *VaR* is available in the literature [9] [10] [11] [12] [13] [14]. Each of the techniques and models relies on a set of assumptions of its own. However, the most common assumption is that the best estimator for future changes in market conditions is the historical trace of market data. Some of the well-known models for estimating *VaR* include:

1. *Variance-Covariance (VCV) model* – It assumes that the risk factor returns are always (jointly) normally distributed and that the portfolio return is normally distributed. It also assumes that the change in portfolio value is linearly dependent on all risk factor returns. The variance-covariance or the delta-normal model was popularized by J.P Morgan in the early 1990s. The assumption that the portfolio return is normally distributed implies that the portfolio is composed of assets, whose deltas are linear, i.e., the change in the value of the portfolio is linearly dependent on all the changes in the values of the assets. This further implies that the portfolio return is also linearly dependent on all the asset returns and that the asset returns are jointly normally distributed. Further assuming that the only risk factor associated with a portfolio is the value of the portfolio itself, the 95% confidence level *VaR* for *N* assets over a holding period, is given by

$$VaR = -V_p(\mu_p - 1.645\sigma_p) \quad (3)$$

where, the mean μ_p is given by

$$\mu_p = \sum_{i=1}^{N} \varpi_i \mu_i \quad (4)$$

and, the standard deviation σ_p is given as

$$\sigma_p = \sqrt{\Omega^T \Sigma \Omega} \quad (5a)$$

$$\Omega = \begin{bmatrix} \varpi_1 \\ \varpi_2 \\ \varpi_3 \\ . \\ . \\ \varpi_N \end{bmatrix} \quad (5b)$$

$$\Omega^T = [\varpi_1 \quad \varpi_2 \quad \varpi_3 \quad . \quad . \quad \varpi_N] \quad (5c)$$

Here, *i* refers to the return on asset *i* and *p* refers to the return on the portfolio for standard deviation (σ_p) and mean (μ_p). V_p is the initial value of the portfolio (in currency units). ϖ_i is the ratio of V_i to V_p. Σ is the covariance matrix between all the *N* asset returns.

The benefits of the *VCV* model are the use of a more compact and maintainable data set, which can often be bought from third parties, and the speed of calculation using optimized linear algebra libraries. Drawbacks include the assumption that the portfolio is composed of assets whose delta is linear and the assumption of a normal distribution of market price returns/asset returns.

2. *Historical Simulation (HistSim) model* – It has emerged as the industry standard for computing *VaR*. This model is based on the assumption that the asset returns in the future will have the same distribution as they had in the past (historical trace). *HistSim* is the simplest and most transparent method of calculation. This involves running the current portfolio across a set of historical price changes to yield a distribution of changes in portfolio value, and computing a percentile (*VaR*). The benefits of this method are its simplicity of implementation. Added to it, it does not assume a normal distribution of asset returns like the *VCV* model. The main drawbacks are the requirement for a large market database and the computationally intensive calculation.

In *HistSim*, *VaR* is evaluated as:

$$VaR = 2.33M\sigma_p\sqrt{10} \quad (6)$$

where, M is the market value of the portfolio, σ_p is the historical volatility of the portfolio. The constant 2.33 stands for the number of σ_p needed for a certainty level of 99% and the constant $\sqrt{10}$ refers to the number of days in the holding period.

Basically, the *HistSim* method computes *VaR* in two simple steps. First, a series of pseudo-historical portfolio returns are constructed using today's portfolio weights and historical asset returns. Second, the quantile of the pseudo-historical portfolio returns is computed to yield *VaR* and the current asset returns.

3. *Monte Carlo simulation* – It randomly simulates future asset returns. Monte Carlo simulation is generally used to compute *VaR* for portfolios containing securities with non-linear returns because the computational effort required is non-trivial. Monte Carlo simulation is conceptually simple, but is generally computationally more intensive than both the *VCV* and *HistSim* models. The generic Monte Carlo *VaR* calculation comprises the following steps:

1. Predefine *N*, the number of iterations to perform.
2. For each iteration in *N*,
   - Generate a random scenario of market moves using some existing market model.
   - Revalue the portfolio under the simulated market volatility scenario.
3. Compute the portfolio profit or loss (*PnL*) under the simulated scenario. For doing so, subtract the current market value of the portfolio from the market value of the portfolio computed in the previous step.
4. Sort the resulting *PnLs* to obtain the simulated *PnL* distribution for the portfolio.
5. *VaR* at a particular confidence level is then calculated using the percentile function.

V. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (*PSO*) is a biologically inspired evolutionary computing paradigm. It is a population-based stochastic optimization procedure inspired by the sociocognitive behavior of bird flocking or fish schooling and was developed by Kennedy and Eberhart [15, 16, 17] in 1995. The original intent was to graphically simulate the choreography of a bird flock or fish school. However, it was found that particle swarm model can be used as an optimizer [17]. The optimization technique starts with a population of random solutions. It searches for an optimum by iterating through generations. But, unlike other evolutionary algorithms such as the genetic algorithms, *PSO* does not employ any evolution operators like crossover and mutation. In *PSO*, the potential solutions are called particles, which fly through the problem space by following the current optimum particles [17].

During the algorithm, each particle keeps track of its coordinates associated with the best solution (fitness) it has achieved so far in the problem space [17]. This stored fitness value is referred to as the *pbest*. Another "best" value obtained so far by any particle in the neighbourhood of the particle is also tracked by the particle swarm optimizer. This is referred to as the *lbest* [18]. When a particle incorporates the entire population as its topological neighbours, the best value is a global best and is called the *gbest*.

The local version of *PSO* comprises, at each time step, changing the velocity of (accelerating) each particle toward its *pbest* and *lbest* locations [17]. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward *pbest* and *lbest* locations [17, 18].

The performance of each particle in a *PSO* is determined by its closeness from the global optimum. The metric commonly used for this purpose is the fitness function of the optimization problem. Each particle in the swarm possesses (i) the *current position* of the particle, (ii) the *current velocity* of the particle and (iii) the *personal best position* of the particle.

PSO has been successfully applied in many application areas due to its ability of getting better results in a faster, cheaper way compared with other methods [17]. Moreover, *PSO* requires only a few parameters to adjust.

A. *The PSO algorithm*

In *PSO*, the swarm comprises a set of particles $P= p_1, p_2, \dots p_k$ [17]. An objective function f represents a candidate solution of the optimization problem at hand. This corresponds to the position of a particle. At a time t, p_i has a position $\bar{x}_i^t$ and a velocity $\bar{v}_i^t$ associated to it [17]. The particle's personal best position that particle p_i (with respect to f) has ever visited until time step t is represented by vector $\bar{b}_i^t$. In addition, p_i receives information from its neighborhood $N_i \subseteq P$. In the standard *PSO* algorithm, the population topology of the swarm is represented by a graph $G = \{V, E\}$, where each vertex in V corresponds to a particle in the swarm and each edge in E indicates a relation between a pair of particles [17].

An initialization region $\theta' \subset \theta$ is chosen to generate the random positions for the particles in the PSO algorithm [17]. Velocities are usually initialized within θ'. However, velocities can also be initialized to zero or to small random values to prevent particles from leaving the search space during the first few iterations [17].

During the algorithm, these velocities and positions of the particles are iteratively updated until a stopping criterion is met [15]. The update rules are [17]:

$$\bar{v}_i^{t+1} = w\bar{v}_i^t + \emptyset_1 \bar{U}_i^t \left(\bar{b}_i^t - \bar{x}_i^t\right) + \emptyset_2 \bar{U}_2^t (\bar{l}_i^t - \bar{x}_i^t) \qquad (7)$$

$$\text{with } \bar{x}_i^{t+1} = \bar{x}_i^t + \bar{v}_i^{t+1} \qquad (8)$$

where w is a parameter called inertia weight [17], ϕ_1 and ϕ_2 are two parameters called acceleration coefficients [17], $\bar{U}_i^t$ and $\bar{U}_2^t$ are two $n\times n$ diagonal matrices in which the entries in the main diagonal are random numbers uniformly distributed in the interval [0,1] [17]. These matrices are regenerated at each iteration. The vector $\bar{l}_i^t$ is referred to as the neighbourhood best. It is the best position ever found by any particle in the neighborhood of particle p_i, i.e., $f(\bar{l}_i^t) = f(\bar{b}_j^t)\ \forall\ p_j \in N_i$ [17].

For properly chosen values of w, ϕ_1 and ϕ_2, the particles' velocities do not grow to infinity [19]. In the *lbest* [18] model, a swarm is divided into overlapping neighborhoods of particles and the best particle is referred to as the neighborhood best particle [17]. There may be various neighborhood configurations [20] in *PSO* depending on particle indices or topological configurations. It is clear that *gbest* is a special case of *lbest* with $l = s$, where s is the swarm size. It may be noted that the *lbest* approach results in a larger diversity; however, it is slower than the *gbest* approach.

The three terms in the velocity-update rule influence the local behaviours of the particles [17]. The first term, referred to as the inertia or momentum term [17, 21], serves as a memory of the previous flight direction. It also prevents the particle from drastically changing direction in the near future. The second term, called the cognitive component term, models the intent of the particles to return to the previously found best positions [17]. The third term, called the social component term [17, 22, 23], quantifies the performance of a particle relative to its neighbours, thereby representing a standard that should be attained.

It has been observed that in some cases, particles can be attracted to regions outside the feasible search space θ [17]. Engelbrecht [24] devised mechanisms for preserving solution feasibility and proper swarm operations for this purpose. One of the alluring mechanisms for preserving feasibility is one in which the particles going outside θ are not allowed to improve their personal best position. In such cases they are attracted back to the feasible space in subsequent iterations [17].

VI. RESULTS

The optimization of the portfolio asset allocation problem has been addressed with the help of a particle swarm optimization technique. From the objective of faithful asset allocation at a given confidence level, the problem boils down to the minimization of the Value-at-Risk (*VaR*) of the portfolio. The *HistSim* model has been used for the estimation of *VaR*. Equation (6) has been employed for this purpose. Moreover, this function has been used as the fitness function in the particle swarm optimization procedure. The parameters used for the particle swarm optimization procedure are given in Table I.

TABLE I: Particle Swarm Optimization parameters Employed

Sl. No.	PSO Parameter	Values used
1.	Number of Generations	(500, 1000)
2.	Inertia weight	0.8
3.	Acceleration Coeffient (ϕ_1)	1.5
4.	Acceleration Coeffient (ϕ_2)	1.5

The portfolio asset allocation optimization procedure has been demonstrated on a collection of 20 portfolios with several asset combinations. The optimization procedure using the particle swarm optimization technique has been run with two different numbers of generations viz., 500 and 1000 with the constants as indicated in Table I. The average of the best fitness results are archived and reported. Table II lists the different archived average optimized portfolios over two different number of generations, their costs and *VaR* measures for a confidence level of 95%.

TABLE II: Optimized portfolios with their costs and *VaR*s at a confidence level of 95%

Portfolio No.	Portfolio Cost (in currency units)	Value-at-Risk
1	**10,000.0**	**0.8000000**
2	35,983.1	0.1760900
3	**21,586.5**	**0.6430960**
4	**30,723.6**	**0.9005500**
5	43,256.7	0.4205077
6	25,517.2	0.5756070
7	22,184.6	0.1652210
8	28,642.3	0.0591332
9	**41,275.5**	**0.7441580**
10	49,357.3	0.5116115
11	47,272.3	0.3877810
12	46,233.5	0.3486000
13	**10,499.9**	**0.8381680**
14	22,066.8	0.2657900
15	**19,161.5**	**0.8986830**
16	**43,286.6**	**0.7335820**
17	15,362.6	0.4645280
18	**44,334.0**	**0.7403780**
19	17,311.5	0.2598440
20	13,515.1	0.2357380

From the table, it is evident that only certain combinations of assets in some portfolios have their *VaR* within the permissible range as decided by the tolerance factor 0.9. These permissible portfolios P_k, k = {**1, 3, 4, 6, 9, 13, 15, 16, 18**}, are depicted in **boldface** in Table II. Hence, it is clear that the optimization process is able to select those portfolios with the permissible loss or *VaR*.

Table III demonstrates another example application of the proposed portfolio optimization based asset allocation approach. Here, the portfolio optimization procedure is carried out subject to a certainty rating of 52.0604%, which signifies the gradation of a particular portfolio in terms of the certainty in achieving a desired return over the holding period.

TABLE III: Ordered portfolios according to optimizations based on minimum *VaR* and certainty rating of 52.0604%

Portfolio No.	Portfolio Cost (in currency	Value-at-Risk

	units)	
1	**2,375.00**	**0.496709**
2	**2,433.12**	**0.508864**
3	**2,491.58**	**0.512091**
4	2,492.19	0.521218
5	2,496.09	0.522035
6	2,498.02	0.522437
7	2,499.51	0.522750
8	2,499.79	0.522807
9	2,499.88	0.522826
10	2,499.94	0.522839
11	2,500.00	0.522839
12	2,500.00	0.522852
13	2,500.00	0.522852
14	2,500.00	0.522852
15	2,500.00	0.522852
16	2,500.00	0.522852
17	2,500.00	0.522852
18	2,500.00	0.522852
19	2,500.00	0.522852
20	2,500.00	0.522852

This example also employs 20 different portfolios with several asset combinations. The corresponding portfolio costs and their optimized *VaR* measures are also shown in the table. The optimized portfolios are listed in Table III in order of increasing *VaR*. From the table, it is clear that out of participating 20 different portfolios, only three portfolios marked in **boldface** are eligible and stand in good stead given the certainty rating of 52.0604%.

From the results obtained in Tables II and III, it is evident that a faithful selection/allocation strategy of assets in a portfolio can be derived at by resorting to an optimization procedure. The beauty of the optimization procedure is that it takes into account several constraints in the form of the certainty ratings or confidence levels thereby ensuring an effective optimum portfolio management in terms of minimum expected losses.

VII. Discussions and Conclusion

Portfolio management has assumed paramount importance as a systematic discipline in the fields of economics and finance given the diversification in investment strategies. This article attempts to evolve a selection and allocation strategy of portfolios in a volatile market condition by means of the optimization of the Value-at-Risk (*VaR*) measures of the portfolios under consideration. A particle swarm optimization procedure is adopted on historical portfolio data for this purpose. Faithful selection results are exhibited on a collection of 20 different portfolios with several asset combinations.

The proposed approach aims at minimizing the *VaR* measures of the portfolios under consideration. Methods however remain to be investigated to incorporate the aspect of return maximization in the portfolio allocation scenario through multiobjective optimization techniques. The authors are currently engaged in this direction.

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